Managing Risk in Multi-Asset Class, Multimarket Central Counterparties: The CORE Approach☆,☆☆

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Abstract

Multi-asset class, multimarket central counterparties (CCPs) are becoming less uncommon as a result of merges between specialized (single-asset class, single market) CCPs and market demands for more capital efficiency. Yet, traditional CCP risk management models often lack the necessary sophistication to estimate potential losses relative to the closeout process of a defaulter’s portfolio in a multi-asset class, multimarket environment. As a result, multi-asset class, multimarket CCPs usually rely upon a simplified silo approach for calculating risk that, not only fails to deliver efficiency, but can also increase systemic risk. The CORE (Closeout Risk Evaluation) approach, on the other hand, provides the conceptual and mathematical tools necessary for robust and efficient central counterparty risk evaluation in multi-asset class and multimarket environments, acknowledging the portfolio dynamics involved in the closeout process, as well as important “real life” market frictions.

Keywords:

☆The CORE approach was originally developed by BM&Bovespa and is currently undergoing an IP registration process.
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1The ideas presented here do not express any private institution view and are solely related to the authors’ personal opinion.
1. Introduction

Clearinghouses have long contributed to the financial services industry by handling, in an efficient and safe manner, the daunting amount of settlement transactions resulting from trading activity in a number of different asset classes. From the institutions that supported the flourishing 18th century banking industry in London, as presented in [14], to the global infrastructure providers that successfully handled the 2008 crisis, the number of clearinghouse failures have been exceptionally small, both in nominal and relative terms, especially when compared to banks and insurers, for instance.

One important service provided by many clearinghouses, besides all activities related to centralized clearing and settlement, is performing the role of central counterparty (CCP) for all cleared transactions. Central counterparties are commonly defined as financial market infrastructures (FMIs) that interpose themselves between counterparties to contracts traded in one or more financial markets, becoming the buyer to every seller and the seller to every buyer and thereby assuring the performance of open contracts, as detailed in [6]. As a result, clearinghouse participants exchange multiple, heterogeneous credit relationships (i.e. risks) for a single, homogeneous credit risk presented by the CCP.

Benefits from employing a CCP model for a given asset class range from avoiding gridlocks and unwind procedures in the settlement process to enhanced price discovery mechanisms due to the single credit risk feature. Yet, systemic risk reduction is unquestionably the most important single value offered by CCPs as a consequence of netting, collateralization and orderly closeout procedures. Those procedures are designed to eliminate or, at least strongly mitigate, the impacts of a participant default, avoiding thus settlement disruptions and disordered resolution processes. Empirical studies, as for instance [15], and the literature on this subject, as [1], maintain that the emergence of systemic risk is often related to specific characteristics, such as size, leverage, concentration and interconnectedness. Although not directly

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2There is, however, some debate concerning the specific conditions in which systemic risk is reduced by the introduction of a CCP. See [4] and [9].
exposed to market risk, as its net position on every cleared contract is, by
definition, equal to zero, CCPs can be exposed to sizable positions in the case
of a participant default. Hence the necessity to maintain robust safeguard
structures, capable of dealing with one or more (simultaneous) participant
default, even throughout extremely adverse market circumstances.

One key element of a typical CCP safeguard structure consists of mar-
gin deposits, that is, assets and cash (i.e. collateral) that participants post
in the CCP to cover their losses should they fail. Margin methodologies
experienced a rapid increase in complexity during the turn of the 20th cen-
tury, evolving from purely ad hoc fixed performance bonds approaches to
risk based portfolio margining. This evolution essentially reflected the need
to clear new products that were not only more complex, but also contingent
to one or more underlying assets. Yet, despite the increasing complexity
of cleared products, clearinghouses tended to cling on specific asset classes,
such as equities, financial derivatives, energy derivatives and so on. This
characteristic reflected both their specialization on certain markets and their
particular ownership structures, where large financial intermediaries where,
at the same time, participants and owners in a mutualized organization.

However, the demutualization movement that, from a practical stand-
point, began in 1992 with Deutsche Börse, was followed by an IPO wave that
reshaped the ownership and governance structures of major global clearing-
houses. This also set the basis for clearinghouse merges, turning specialized
(single-asset class, single market) clearinghouses into multi-asset class, mul-
timarket environments. An exhaustive account on the evolution of the clear-
inghouse landscape can be found in [16]. This is the context now and there
is no clear evidence that consolidation movements have exhausted their mo-
mentum, specially taking into account renewed regulatory pressure for more
collateralized transactions and, as a result, market demands for efficient cap-
ital allocation.

Margin methodologies have now to take a new leap forward in order to
deal with this Brave New World of multi-asset class, multimarket CCPs.
Yet, this is not as simple as revamping existing models, enhancing them in
order to be able handle new asset classes and markets. Indeed, mainstream
methodologies, more often than not, are asset class and market specific, able
only to manage homogeneous portfolios, that is, portfolios in which assets
have similar risk profiles. In fact, estimating potential losses relative to the
closeout process of a defaulters portfolio when this portfolio comprises mul-
tiple asset-classes and markets (i.e. is highly heterogeneous), pose a number
of challenges in terms of risk modeling that, if not correctly addressed, can give rise to important margin shortfalls.

The objective of this paper is to present a risk approach that estimates potential losses relative to the closeout process of a defaulter’s portfolio in multiple asset-class, multimarket CCPs. This approach or methodology, named CORE (Closeout Risk Evaluation), takes into account important differences between distinct asset classes and markets that have a direct impact on risk estimates. Thus, Section 2 presents the CORE approach; evidencing its differences from mainstream, single-asset class, single market methodologies. Sections 3 to 5 describe the main components of the methodology, namely the definition of the optimal closeout strategy, the estimation of potential losses given the closeout strategy and the c-value calculation. Section 6 shows a practical example and Section 7 concludes.

2. The CORE Approach

The vast majority of the risk methodologies currently employed by the financial industry can trace their origins back to a common forefather: the value-at-risk approach, VaR, brought into light in early 90s by J. P. Morgan with the publication of the RiskMetrics technical document, whose details may be obtained on the classical reference [11] and [12], or with some adaptations in [3]. One of its most appealing attributes was the ability to encapsulate the market risk of a given portfolio into a single number that represents the potential loss considering a fixed holding period and a given confidence level. This “single risk figure” concept had a strong influence on the development of traditional CCP risk management models, as it could easily be translated into a margin requirement. Thus, besides differences in the way that the future states of the world were represented (Monte Carlo simulations vs. parametric distributions vs. historical data vs. stress testing, for instance) and risk calculated (full valuation vs. linear approximations, for instance), the “single risk figure”/margin equivalence became the norm.

An important caveat, though, is that the “single risk figure” represents a loss in market value considering the whole portfolio under analysis. In other words, it pictures the same portfolio in two different environments (today and future) and calculates the difference (loss) in terms of market value. Transposed to the CCP world, that means that all defaulter’s positions are going to be reversed (closed out) at the same time and, consequently, in the same market scenario. Although this can be a fairly good proxy for the
potential losses relative to the closeout process of a defaulter’s portfolio in the case where asset classes that are similar, this is not necessarily true for highly heterogeneous portfolios. Differences concerning market rules, trading mechanisms, price discovery processes, liquidity profiles, fungibility, cash flow structures, convertibility, settlement procedures and the like make it exceedingly hard to compute coherent risk estimates given just two points in the time continuum.

This problem is usually circumvented using a silo approach, where similar assets are grouped together in silos that do not communicate with each other, so the total risk is given by the sum of the risks of each individual silo. Also, sometimes a very simple ad hoc “credit spread” allows for some risk offsetting (i.e. margin relief) between different silos. Unfortunately, simply avoiding risk underestimation does not necessarily mean reducing systemic risk in the CCP context. Failure to capture hedges between different asset classes and markets in the silo approach means heftier margin requirements for protected positions relative to outright, riskier, portfolios. Therefore, besides not giving the right incentives to hedge positions across different asset classes and markets, the silo approach can also increase systemic risk by escalating margin requirements for hedged portfolios during adverse market conditions. Taken to the limit, this means that a participant could collapse due to increasing margin requirements, albeit being hedged.

The CORE risk management framework was specifically developed to tackle the problem of estimating central counterparty risk in multi-asset class, multimarket environment, overcoming the “single risk figure”/“multiple-silos” model deficiencies described previously. A detailed comparison of the approaches, alongside with some other important features of CORE, may be obtained at [5]. Figure 1 illustrates the differences between the CORE, the “single risk figure” and the “multiple silos” approaches concerning the problem of estimating central counterparty risk in multi-asset class, multimarket environments.

[FIGURE 1: A.1]

The single most important feature of the CORE framework consists in its ability to recognize that closeout processes are dynamic, so the portfolio’s risk profile changes as positions are settled and/or reversed through time. Recognizing that default management processes are dynamic allows the model to define a closeout strategy that takes into account important trading, settlement and liquidity constraints. For instance, a given position on asset A
can bet reverted on $t^*+1$, whereas its mirror/hedge position on asset B can only be reverted on $t^*+2$ due to a local holiday. Or a given OTC position C does not admit partial liquidation, whereas its mirror/hedge position in listed futures is not subject to this restriction. Thus, selecting the appropriate closeout strategy entails defining the closeout sequence (that is, which asset, at which amount and at what time) that minimizes the losses associated to the closeout process, while respecting, at the same time, trading, settlement and liquidity constraints.

The second component of the CORE approach is to estimate the changes in market value for all assets considered, taking also into account the portfolio dynamics established by the closeout strategy. That, of course, can be accomplished in a number of different ways. One might prefer, for instance, to re-price all assets during the closeout period using Monte Carlo simulations, rather than using historical prices for equivalent time frames. The important result, though, is the establishment of a P&L structure containing, for each state of the world (i.e. risk scenario), the financial value stream (i.e. on $t^*+1$, $t^*+2$, ..., $t^*+T$) for every position.

The P&L structure can be conveniently aggregated into a set of cash flow structures for each state of the world, allowing the determination of a series of consolidated risk metrics. Adding up all cash flows for a given state of the world yields the cost of closing out the portfolio in that specific scenario. Ordering the aforementioned results allows for defining a convenient closeout risk metric, such as the 99th worst case scenario, the 95th worst case scenario shortfall risk or the absolute worst case scenario. Yet, there is also another important risk metric that can be derived from the cash flow structures, that is the maximum liquidity (funding) need during the closeout process. The importance of this figure cannot be emphasized enough, as clearinghouses must have the means to cope with the default management process as a whole, which might entail funding needs that are far superior to the expected total loss given the closeout process.

For instance, suppose for given portfolio the worst case scenario is a negative cash flow of $100M on $t^*+1$ and a positive cash flow of $90M on $t^*+3$. In this case, the total loss given the closeout process is -$10M (-$100M+$90M). However, there is a substantial need for funding ($100M) from $t^*+1$ to $t^*+3$ that should be either be backed by liquidity mechanisms or liquid collateral (i.e. cash or assets that are cash equivalent). Accordingly, the CORE approach defines two closeout risk metrics: the total permanent loss and the total transient loss. The total permanent loss reflects the net cost of closing
out a defaulter’s portfolio, whereas the total transient loss reflects the extra funding need in excess of the total permanent loss.

Figure 2 illustrates the three key components of the CORE framework: definition of the optimal closeout strategy, potential loss evaluation and loss metrics calculation. From these metrics the CORE risk measurement c-value is obtained.

[FIGURE 2: A.2]

3. Optimal Closeout Strategy

The first component of CORE methodology is to define the closeout strategy to be considered for the defaulter’s portfolio. A closeout strategy represents a potential structure for liquidating or settling the portfolio. This structure varies accordingly to its objectives, and may be as simple as liquidating all positions of the portfolio as soon as possible, what may be called a naive strategy, or it may engender a more complex format regarding a series of rules to which the strategy must obey. The closer the strategy is to reaching its objectives the better, and the best one from the available set is defined as the optimal closeout strategy.

Following the optimal hedge literature, one possibility for the determination of optimal strategies can be its evaluation with respect to a decision problem, which usually involves a utility function, $U(.)$, and a statistical decision metric. In the groundbreaking papers of the relevant literature, [7] and [8], the authors propose that, in the case of the search for the perfect hedge of a portfolio, the problem could be expressed as $\max_{\theta \in \Theta} E[U(W)]$, in which the mathematical expectation, $E[.]$, represents the decision metric, $W$ expresses the wealth or value of the portfolio and $\theta$ the quantity of the hedge. Notice that the decision of the strategy $\theta$ is taken against the set of all available possibilities $\Theta$. The strategy which solves this problem, $\theta^*$, is called the optimal one. For more contemporary developments on the literature see the work of [10], and references cited there.

The idea of establishing perfect hedges for a portfolio set forth in the optimal hedge literature will be the fundamental driving premise underlying the process for obtaining the optimal closeout strategy in the CORE methodology. In this regard, the decision problem framework of the latter approach can be extended to CORE, accounting for the distinction between a theoretical model for explaining the agents portfolio choice and the specificities that
a clearinghouse activity possesses. Accordingly, in broad terms, the objective subjacent to the CORE decision problem is to create a closeout strategy which preserves the natural hedges of the defaulter’s portfolio, minimizing the losses incurred during its liquidation.

3.1. Portfolio and Closeout Strategy

Prior to present the CORE decision problem some preambles are necessary. The first of them is the characterization of portfolio, and most importantly, the closeout strategy associated to it. A portfolio is simply a collection of $I$ financial instruments. Let it be represented by the $I$-tuple $Q_0 = (Q_{1,0}, Q_{2,0}, ..., Q_{I,0})$, in which each $Q_{i,0}$ expresses the total quantity of instrument $i$ at an original evaluating time $t^*+0$. In the CORE framework, this set can be composed not only by the contracts available to trade within the clearing enforcement, but also by the assets pledged as the collaterals to margin these contracts in the CCP.

In what regards the closeout strategy, the first relevant aspect is the time horizon accompanying it. This horizon designates the period necessary to settle/liquidate all the instruments of the portfolio, with its structure and length depending on the composition of the portfolio. For CORE, this horizon is given by the discrete set $T = \{1, 2, ..., T\}$, in which $T$ is the longest time length of the period. Therefore, the matrix $Q_{I \times T}$ defines a closeout strategy, with each element $Q_{i,\tau}$ expressing the quantity of asset $i$ liquidated at time $t^*+\tau$,

$$Q = \begin{bmatrix}
Q_{1,1} & Q_{1,2} & \cdots & Q_{1,T} \\
Q_{2,1} & Q_{2,2} & \cdots & Q_{2,T} \\
\vdots & \vdots & \ddots & \vdots \\
Q_{I,1} & Q_{I,2} & \cdots & Q_{I,T}
\end{bmatrix}.$$  

Instead of working with integer quantities of instruments, a transformation to relative values can be used to facilitate the following calculations. Specifically, it is done according to the simple relation $q_{i,\tau} = \frac{Q_{i,\tau}}{Q_{i,0}}$, and will yield new matrix $q_{I \times T}$.

3.2. Risk Scenarios

The treatment of uncertainty is another relevant feature of CORE methodology. The concept will be modeled as potentials realizations of the states of the nature, or in other terms, futures outcomes of the world. Formally, uncertainty assumes the form of a probability space $(\Omega, \mathcal{F}, P)$. Differently
than most of the papers in optimal hedge literature, this will be a discrete space, with finite states of the nature, \( \omega_m \in \Omega, m = \{1, 2, ..., M\} \).

Measurable on this set there is a series of random vectors, \( S_j(\omega) \) with \( j = \{1, 2, ..., J\} \), each one of them representing the realization of a financial variable along the future time horizon \( T \). In the financial industry \( S_j \) is also called a risk factor. Let the \( T \)-tuple \( S_k(\omega) = (S_{k,1}(\omega), S_{k,2}(\omega), ..., S_{k,T}(\omega)) \) characterize one of these vectors. Then, each element \( S_{k,\tau}(\omega) \) expresses the value of this variable \( k \) at a specific time \( t^*+\tau \). To be more precise, \( S_k(\omega) \) could, for instance, be the FX rate of US dollar per Euro, USD/EUR, and \( S_{k,\tau}(\omega) \) be its predicted value at the future date \( t^*+\tau \).

The matrix \( R(\omega)_{JxT} \) formed by the collection of random vectors and associated to a specific state \( \omega = \omega_k \) is called a risk scenario in the CORE methodology,

\[
R(\omega) = \begin{bmatrix}
S_{1,1}(\omega) & S_{1,2}(\omega) & ... & S_{1,T}(\omega) \\
S_{2,1}(\omega) & S_{2,2}(\omega) & ... & S_{2,T}(\omega) \\
... & ... & ... & ...
\end{bmatrix}.
\]

Basically, a risk scenario maps a state of the nature into a set of realizations of risk factors. There will be as many risk scenarios as the dimension of the probability space \( \Omega \) considered. In order to simplify notation, this dependence from the state of the nature will be omitted from now on, resuming risk scenarios by \( R \), and the set formed by all risk scenarios by \( R \).

Finally, differently than traditional risk measurements, in CORE methodology the domain of the analysis takes places in the cube formed by \( M \) risk scenarios, \( J \) risk factors and \( T \) time horizons, having this last dimension as a new feature.

3.3. Decision Problem

The next step to determine the optimal closeout strategy is the formalization of the decision problem per se. First, it is important to notice that the configuration exhibited here is one of the many possible for determining optimal closeout strategies within CORE methodology, an evidence also signalized in [2]. In this particular setting, the utility function has a high potential to define optimal hedges for market prices variation, correctly measuring the potential losses of the closeout strategy. Therefore, the function will depend on the strategy itself, \( U(q) \), given by the \( q \)-tuple derived from the \( q \) matrix. Specifically, it can be expressed by
\[ U(q) = L(q, R) = \frac{1}{T} \cdot \sum_{\tau=1}^{T} L(q_{\tau}, R, \tau), \] (1)

in which \( q_{\tau} \) is a subset of the \( q \)-tuple only containing data for time horizons smaller or equal to \( \tau \).

In order to replicate the potential deficits of the strategy, the loss function \( L(q_{\tau}, R, \tau) \) may be segregated into two parts, the realized loss and the unrealized one,

\[ L(q_{\tau}, R, \tau) = L_r(q_{\tau}, R, \tau) + L_u(q_{\tau}, R, \tau). \] (2)

As it will become clear throughout the paper, the distinction between them basically refers to the settlement decision and, consequently, how it translates into potential losses. While the former express the cost of liquidating the positions until time \( t^*+\tau \), the unrealized loss seeks to capture the price for carrying them until that time. Indirectly, they respectively also reflect potential liquidation flows \textit{vis-a-vis} mark to market variation.

The final step in order to precisely determine the utility function is to present how it relates to the instruments of the portfolio. To proceed in this direction it is necessary to rewrite \( q_{i,\tau} \) in terms of a balance variable, \( q_{i,\tau} = s_{i,\tau} - s_{i,\tau+1} \), in which \( s_{i,\tau} \) represents the balance of instrument \( i \) at time \( t^*+\tau \). Therefore, the realized and unrealized loss functions may be expressed by

\[ L_r(q_{\tau}, R, \tau) = \sum_{t=1}^{\tau-1} \sum_{i=1}^{I} (s_{i,t} - s_{i,t+1}) \cdot \phi_i(R, t), \] (3)

\[ L_u(q_{\tau}, R, \tau) = \sum_{i=1}^{I} (s_{i,\tau}) \cdot \phi_i(R, \tau). \] (4)

In these expressions the function \( \phi_i(R, \tau) \) summarizes the variation on the financial position held on the referred instrument. Specifically,

\[ \phi_i(R, \tau) = Q_{i,0} \cdot [P_i(R, \tau) - P_i(0, 0)], \] (5)

in which \( Q_{i,0} \) is the figure for the total quantity of instrument \( i \); \( P_i(R, \tau) \) is the unitary price function for instrument \( i \) evaluated at risk scenario \( R \) and time \( t^*+\tau \); and \( P_i(0, 0) \) refers to the unitary market price of instrument \( i \) at time \( t^*+0 \).
An important aspect of the $\phi(R, \tau)$ is that it can be applied to various sets of financial instruments and strategies held on them. If the instrument does not have variation margin, then the function simply express the fluctuation on its price from the market value to a stress scenario on a future date. Conversely, for those instruments that mark to market on a period basis, possessing no intrinsic value at the end of this time interval, the function approximates the respective accumulated margin call. Besides that, for each type of instrument the function assumes a different form, ultimately mirroring the way the market values them. Finally, it works for both long and short position, only modifying the sign of $Q$.

In order to formulate the decision process not only the utility function is required. Another part is the statistical decision metric to be considered when evaluating all the potential realizations of the states of the nature, $\Omega$, or its mappings to risk scenarios, $R$. While on the studies cited above the mathematical expectation is employed, for the present case the minimum function will be preferred. Based on this selection, the problem kernel could be expressed by $\min_{R \in R} L(q, R)$. In particular, the selection of the minimum has two classes of justifications, one based on managerial risk principles and another on the mathematical properties of the function desirable to its implementation.

From the point of view of a central counterparty activity, the usage of the mathematical expectation may be too optimistic. There may exist cases in which the hedge generates severe losses for which the CCP must not be willing to pay. Given its systemic role, the closeout hedge to be adopted must be sufficiently robust to minimize the losses in all space of states, but more importantly in the worst envisaged scenario. If it is efficient for this scenario, then it will also be for the remaining less austere ones. Regarding its calculation properties, as discussed in [13], the minimum function, over a discrete domain, is piecewise linear and concave. The most important implication refers to the fact that maximizing it, subject to a set of linear constraints, can be transformed into a linear programming problem, for which standard procedures are available.

From the discussion above, it is possible to present the whole decision problem applied by the CORE methodology to establish the optimal closeout strategy,

$$\max_s \min_{R \in R} L(s, R),$$

(6)
s.t.

\[
0 \leq s \leq 1, \\
s_{i,\tau} = 1, 0 \leq \tau < \tau_{i}^*, \\
(s_{i,\tau} - s_{i,\tau+1}) \leq \kappa_{i}.
\]

The restrictions basically refer respectively to: i) the only tradable amounts are the originals ones; ii) there exists a minimum period of time to start negotiating each instrument \(i\), given by the parameter first day to trade, \(\tau_{i}^*\) and; iii) there is a maximum proportional quantity of the instrument \(i\) that can be traded each day without impacting market prices, \(\kappa_{i}\), called maximum amount to trade parameter. Although calculated regarding the balance amount, \(s\), it can be easily converted into liquidation quantities, as specified above.

With the specified frame for decision problem, it is possible to argument that \(q^*\), which solves it, minimizes losses suffered during the liquidation window. By the way the problem is constructed, the optimal closeout strategy obtained in its solution is robust with respect to price variations, the elasticity of the market and to some extent, financial flows mismatching. In this regard, the optimal strategy is the output of decision problem which seeks to minimize market risk and liquidity risk, employing hedges and restrictions as its means.

4. Potential Loss Evaluation

The establishment of the optimal closeout strategy represents the first component of the CORE methodology. Upon this liquidation strategy it is necessary to measure the gains and losses that it generates for each one of the risk scenarios, \(R \in \mathbb{R}\). Specifically, conditional on a given \(R\), these measures will be taken for each instrument \(i\) and time horizon \(\tau\), resulting in a matrix \(V_{I\times T}(R)\) of financial values \(\nu_{i,\tau}(R)\),

\[
V(R) = \begin{bmatrix}
\nu_{1,1}(R) & \nu_{1,2}(R) & \ldots & \nu_{1,T}(R) \\
\nu_{2,1}(R) & \nu_{2,2}(R) & \ldots & \nu_{2,T}(R) \\
\vdots & \vdots & \ddots & \vdots \\
\nu_{I,1}(R) & \nu_{I,2}(R) & \ldots & \nu_{I,T}(R)
\end{bmatrix}.
\]

The purpose of these values is to synthesize actual revenues and losses obtained during liquidation time horizon under a specific realization of a state
of the nature. Associated to the portfolio there will be \( M \) matrixes \( V \), one for each state, or similarly, one for each risk scenario \( R \).

The values of \( \nu_{i,\tau} \) will be calculated by two new functions \( \psi_{i}^{s}(R, \tau) \) and \( \psi_{i}^{q}(R, \tau) \), depending whether the instrument is being settled or carried from one period to the other. As well as \( \phi(R, \tau) \), their forms are suitable for different sets of instruments. However, differently from the previous one, a distinction exists for instruments with variation margin and without it. Also, these functions may have a subtle modification to adequate to specificities of some kinds of instruments, as in the case when reference prices are quoted in a foreign currency but margin calls are made on nation one.

Regarding the instruments with variation margin, the functions are expressed as

\[
\psi_{i}^{s}(R, \tau) = Q_{i,0} \cdot [P_{i}^{s}(R, \tau) - P_{i}(R, \tau - 1)],
\]

and

\[
\psi_{i}^{q}(R, \tau) = Q_{i,0} \cdot [P_{i}^{q}(R, \tau) - P_{i}(R, \tau - 1)],
\]

with \( Q_{i,0} \) and \( P_{i}(R, \tau) \) defined as above and; \( P_{i}^{q}(R, \tau) \) designed to express the possibility of liquidating at a price different from the reference one. From these, it is possible to determine the financial values

\[
\nu_{i,\tau}(R) = s_{i,\tau}^{*} \cdot \psi_{i}^{s}(R, \tau) + q_{i,\tau}^{*} \cdot \psi_{i}^{q}(R, \tau),
\]

in which \( s_{i,\tau}^{*} \) and \( q_{i,\tau}^{*} \) represent respectively the optimal balance and quantity settled at time \( t^{*} + \tau \).

For the instruments without variation margin the function \( \psi_{i}^{q}(R, \tau) \) will have to be adjusted. These instruments, which can be a stock posted as collateral or a position held on a swap, carry an intrinsic market value that cannot be discarded when evaluating the financial flow of the closeout strategy. In particular, it will assume the following form

\[
\psi_{i}^{q}(R, \tau) = Q_{i,0} \cdot P_{i}^{q}(R, \tau),
\]

in which terms are characterized as before. Therefore, the liquidation values of these instruments are given by

\[
\nu_{i,\tau}(R) = q_{i,\tau}^{*} \cdot \psi_{i}^{q}(R, \tau).
\]
5. Permanent and Transient Loss Measures

The execution of the optimal closeout strategy generates a matrix of gains and losses for each state of the nature represented by a risk scenario, as seen above. In order to resume each one of these into a set of metrics that expresses the realistic potential losses of the strategy two concepts are proposed, the permanent loss and the transient loss measure.

The former figure is formulated to present the final loss that will be incurred when the liquidation of the portfolio is over. In order terms, it will sum the gains and losses for all instruments for all periods of time composing the liquidation strategy. Particularly, let $\nu_\tau(R) = \sum_{i=1}^{I} \nu_{i,\tau}(R)$ expresses the aggregated deficit or surplus value of all instruments for time $t^*+\tau$. Thereby, the permanent loss for a given risk scenario can be defined as

$$PL(R) = \min \left( \sum_{\tau=1}^{T} \nu_\tau(R), 0 \right). \quad (12)$$

Although the permanent loss is a relevant metric for evaluating the result of a closeout strategy, it is not the only one. Another important measure is how much additional resources will be requested in order not to cause a financial mismatch during the liquidation period. To be more precise, even though there is not permanent loss, meaning that the portfolio is not loser at the end, still there may be potential losses associated to a temporary need of capital to offset a punctual negative result. In particular, this additional amount can be defined as

$$ML(R) = PL(R) - \min \left( \nu_1(R), \nu_1(R) + \nu_2(R), ..., \sum_{\tau=1}^{T} \nu_\tau(R), 0 \right). \quad (13)$$

The risk administration policy of the CCP impacts how this temporary necessity can actually be considered a potential loss to be capitalized. In a particular frame, all the liquidity mismatches could be funded within the CCP safeguard structure, releasing the owner of the portfolio from the need to pre-allocate resources to deal with them. In an opposite configuration, called self-financing one, temporary additional resources would be provided by the participant itself, although any remnant amount after the complete execution of the strategy would flow back to him. These possibilities influence transient loss metric through the variable $\Lambda(ML)$, which determine how much of the
temporal losses will be capitalized by the portfolio’s owner. Therefore, transient loss is given by

\[ TL(R) = \min(-ML + \Lambda(ML), 0). \] (14)

The sum of permanent and transient metrics for each one of the risk scenarios considered measures the total potential loss associated to it. In order to establish the risk embedded in a portfolio, or in specific terms the CORE methodology risk measure, *c-value*, it is then necessary to resume the total potential losses into a single measurement. In particular, it is simply done resorting to the scenario which minimizes the potential losses of the portfolio,

\[ c-value = \min_{R \in \mathbb{R}} (PL(R) + TL(R)). \] (15)

Although other forms of summarization could be considered, the *c-value* is the relevant proposed metric for initial margin calls to the owner of the portfolio, when the collateralizing model is defaulter’s pay.

6. The CORE Approach: an example

In order to exemplify the CORE methodology, suppose a portfolio composed by three types of instruments, an Interest Rate Swap (SDP), an Interest Rate Future (DI1) and an equity (XYZA), respectively with a long and a short position on the first two. The SDP is a kind of a Fixed Floating Swap, having no intermediary coupon payments. It is an OTC contract, with restricted liquidity. The DI1 represents a listed contract, in which investor trades compounded one-day future interest rates, with daily variation margin. It exhibits high liquidity for most representative maturities. Finally, the XYZA expresses a generic listed equity, tradable in the electronic platform, posted as collateral in the particular portfolio. In order to facilitate, it can be considered also liquid.\(^3\)

The portfolio and the parameters may be seen in the Table 1. Although hypothetical, they intend to evidence some relevant features of the methodology. An initial one is the parameter first day to trade, for which SDP exhibits a higher value when compared with the other two instruments. The

\[^3\text{Interested reader is invited to access the website www.bm&fbovespa.com.br for further details on the instruments.}\]
<table>
<thead>
<tr>
<th>Instrument</th>
<th>Quantity</th>
<th>Notional</th>
<th>Days to Maturity</th>
<th>$\tau^*$</th>
<th>$\kappa$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SDP</td>
<td>1</td>
<td>2,000,000</td>
<td>126</td>
<td>4</td>
<td>n.a.</td>
</tr>
<tr>
<td>DI1</td>
<td>-10</td>
<td>100,000</td>
<td>126</td>
<td>1</td>
<td>10,000</td>
</tr>
<tr>
<td>XYZA</td>
<td>40</td>
<td>20</td>
<td>n.a.</td>
<td>1</td>
<td>10,000</td>
</tr>
</tbody>
</table>

Table 1: Portfolio Configuration - $\tau^*$ represents the first day to trade parameter, $\kappa$ represents the maximum amount to trade parameter and n.a. stands for non-applicable.

Heterogeneity rests on the fact that, while listed instruments are fungible and can be liquidated with a conventional order to the matching engine, OTC contracts carry specificities that are difficult to rehearse. A more delicate settlement process is requested by these contracts, reflecting on a longer date to trade. Besides that, in most cases, liquidation will be done all at once, with no possibility for fractionated execution.

In order to proceed, it is necessary to establish the risk scenarios to be considered. However, to do so, the risk factors have to be primarily determined. This can be done resorting to DI1 future contract reference price and to the discrete non-arbitrage formula of the swap, given respectively by

$$PU = \frac{100,000}{(1 + r)^{\frac{D}{252}}}.$$  \tag{16}$$

$$MtM = N \cdot \left[ \frac{(1 + r^*)}{(1 + r)^{\frac{D}{252}}} - \prod_{d=1}^{P} \frac{(1 + r_d)}{(1 + r)^{\frac{D}{252}}} \right],$$  \tag{17}$$
in which $N$ is the notional amount, $r$ represents the one-day interest rate compounded until expiration annually formatted, $r^*$ expresses the fixed interest rate traded to the period, $r_d$ the one-day interest rate measured $d$ days away from the evaluation time and $D$ the number of business days until maturity.

In traditional single period analysis, only one risk factor is considered in these formulas, $r$. At the evaluating time, the other variables are taken as granted, and simplification exists between appreciation and discount rates. In the CORE framework, in which liquidation occurs over time, another risk factor must be considered, $r_d$. This overnight rate has to be predicted for the liquidation horizon. Regarding the portfolio, besides this two, an additional risk factor has to be contemplated, the price of the equity posted as collateral, $S$. 

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Suppose, in an educative view, that there are two possible states of the nature, bullish (U) and bearish (E). Regarding the risk factors of the portfolio and its liquidation horizon, the two potential risk scenarios are of the following form, having in its first row the values for $r$, on the second for $r_d$ and on the third for $S$. The first two rows are expressed in percentage terms, and the last in price units. Also, the first column represents the market reference data at time $t^*+0$, while the following ones respectively $t^*+1$, $t^*+2$, $t^*+3$ and $t^*+4$

$$
R(U) = \begin{bmatrix}
7.25 & 6.28 & 6.31 & 6.34 & 6.37 \\
0.030 & 0.025 & 0.025 & 0.026 & 0.026 \\
20.00 & 22.00 & 22.22 & 22.44 & 22.66 \\
0.030 & 0.035 & 0.035 & 0.036 & 0.036 \\
20.00 & 18.00 & 17.91 & 17.82 & 17.73
\end{bmatrix}
$$

$$
R(E) = \begin{bmatrix}
0.030 & 0.035 & 0.035 & 0.036 & 0.036 \\
20.00 & 18.00 & 17.91 & 17.82 & 17.73
\end{bmatrix}
$$

It is important to highlight that although $r$ expresses an interest rate to a specific period, with changing length during the liquidation horizon, in the present example it is treated as a unique risk factor. It is a specific simplification device, not related to the term structure of the curve.

Defined the initial inputs, the first component of CORE methodology is the computation of the optimal closeout strategy. The strategy has to be robust enough to minimize losses on all risk scenarios, but especially on the worst case one, in the present example the bearish. Consider first the swap contract, which has an MtM of $\$848.25$ on $t^*+0$. Its first day to trade parameter is set to 4 days. Considering the fact that risk grows with time and that there is no other reason to carry it forward, the ideal period for settlement is $t^*+4$.

The short position on future contracts poses as a natural hedge for the market fluctuation of the swap. Both DI1 and SDP are exposed to the risk factor $r$, although in an opposite way. This hedging role implies that holding the execution of the future contract until $t^*+4$ may be beneficial in terms of the final result from the closeout. On the bearish scenario, while the SDP tend to lose values with time, the DI1 is gaining. Important to notice that the fluctuation of DI1 value generates real flow of resources, not only MtM value as is the case of the swap.

Finally, considering the equity XYZA, the decreasing values of the price with time justify early liquidation. The sooner the closeout the better.
Thereby, the optimal closeout strategy in terms of integer quantities is expressed as

\[ Q = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 10 \\ 40 & 0 & 0 & 0 \end{bmatrix}. \]

The reasoning presented above for the computation of the optimal closeout strategy in words is the same one of that pursued by the decision problem, having the utility function \( U(.) \) as its guidance tool. In fact, it can be verified that the strategy determined above is also optimal when evaluated in terms of the portfolio P&L and the respective loss metrics, not only for the bearish scenario, but also for the bullish one.

The second component of CORE methodology regards the financial analysis of the closeout strategy defined. In particular, let \( V(U) \) and \( V(E) \) be the matrixes of gain and losses for the bullish and bearish scenario

\[
V(U) = \begin{bmatrix} 0 & 0 & 0 & 10,772 \\ -5,719 & 51 & 50 & 47 \\ 880 & 0 & 0 & 0 \end{bmatrix},
\]

\[
V(E) = \begin{bmatrix} 0 & 0 & 0 & -13,791 \\ 6,198 & -315 & 261 & 60 \\ 720 & 0 & 0 & 0 \end{bmatrix}.
\]

From these, it is evident that the worst loss on the bullish scenario is derived from cash flow mismatches, once after time \( t^*+1 \) the portfolio only presents positive entries. On the contrary, for the bearish scenario losses arise primarily from the final negative market value of the swap. Holding the DI1 alleviate part of the decreasing value of the SDP. Although another part is compensated by the equity selling, not all losses can be avoided. As previously stated, these matrixes reaffirms that the bearish scenario is the worst one.

The third, and final, component of the CORE methodology refers to the determination of \( c\)-value, which results from the permanent and transient loss metrics evaluated at the worst case scenario. In the present example, these values are respectively $-6,866.79 for the permanent and $0 for the transient loss, resulting in a \( c-value = -6,866.79 \). Specifically, this last figure represents the amount of capital that the owner of the portfolio would have to collateralize within the CCP.
7. Final Considerations

One of the key drivers for the merger between the BM&F and Bovespa in 2008 was the possibility of integrating the four clearinghouses managed by both companies into a single entity. That would result into considerable gains of scale and synergies, creating a top notch, exceptionally efficient post-trade environment. So, during the first half of 2010, the new institution, now BM&FBovespa, launched internally its Post-Trade Integration Project, aiming at integrating the equities (spot, securities lending and single name derivatives), derivatives (financial and commodity listed and OTC derivatives), FX (interbank spot FX market) and government bonds (spot, repo and securities lending) clearinghouses.

The development of the CORE framework began in November 2010 as one of the most important and technically challenging components of this project. The fundamental question in the beginning was how to deliver efficiency without losing consistency and robustness? Answering that question involved a number of Gedankenexperiments (thought experiments) concerning the way a clearinghouse would manage a default in its environment. The conclusion was that no static methodology could yield adequate results for minimally heterogeneous portfolios. Considering default management procedures, and thus the closeout process, as a set of actions carried out in the time continuum (i.e. a dynamic process), subject to a number of “real life” market frictions and constraints, on the other hand, allowed for a more accurate image of reality. Hence, risk offsets could be implemented in a way that was consistent with the actual functioning of the markets considered.

As would be expected, there is not a single way of implementing CORE’s three-tiered methodology, so that’s why calling it an approach or a framework is far more appropriate. Even defining the closeout strategy can be achieved by other means than linear optimization - the choice depends fundamentally on the complexity of the portfolios considered and the degree of accuracy desired.

Finally, it is also worth mentioning that, although originally developed with a specific problem in mind (i.e. multi-asset class and multimarket central counterparty risk evaluation), the CORE framework can also find application in other environments that face similar default management problems, such as the prime brokerage industry.
Appendix A.

Figure A.1: Risk Approaches Comparison

Figure A.2: CORE Components
References


