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Exponencial $[Exp(\lambda),\ \lambda > 0]$

$f(x) = \lambda \exp\{-\lambda x\} \underset{(0,\infty)}{I}(x)$; $\mathbb{E}(X) = \frac{1}{\lambda}$; $\mathbb{E}(X^k) = \frac{k!}{\lambda^k}$; $K(X) = 6$;

$F(x) = 1 - \exp\{-\lambda x\} \underset{(0,\infty)}{I}(x)$; $x_p = \frac{-\ln(1-p)}{\lambda}$; $Var(X) = \frac{1}{\lambda^2}$;

$Mo(X) = 0$; $M_X(t) = \frac{\lambda}{\lambda - t},\ t < \lambda$; $A(X) = 2$; $Med(X) = \frac{\ln 2}{\lambda}$;

- $X \sim Exp(\lambda) \Rightarrow Y = \lambda X \sim Exp(1)$
- $X_1, \cdots, X_n$ i.i.d. com $X_i \sim Exp(\lambda) \Rightarrow Y = \min\{X_1, \cdots, X_n\} \sim Exp(n\lambda)$
- X_1, X_2 i.i.d. com $X_i \sim Exp(\lambda) \Rightarrow Y = X_1 - X_2 \sim La(0, \lambda)$
- $X \sim Exp(\lambda) \Leftrightarrow X \sim Gama(1, \lambda)$
- $X_1, \cdots, X_n$ i.i.d. com $X_i \sim Exp(\lambda) \Rightarrow Y = \sum_{i=1}^n X_i \sim Gama(n, \lambda)$

Weibull I $[W_1(a,b),\ a > 0, b > 0]$

$f(x) = abx^{b-1} \exp\{-ax^b\} \underset{(0,\infty)}{I}(x)$; $Med(X) = (\frac{\ln 2}{a})^{1/b}$;

$Mo(X) = (\frac{b-1}{ab})^{1/b}$; $F(x) = 1 - \exp\{-ax^b\} \underset{(0,\infty)}{I}(x)$;

$Var(X) = a^{\frac{-2}{b}}[\Gamma(1 + \frac{2}{b}) - \Gamma^2(1 + \frac{1}{b})]$;

$x_p = [-\frac{\ln(1-p)}{a}]^{\frac{1}{b}}$; $\mathbb{E}(X^k) = a^{\frac{-k}{b}} \Gamma(1 + \frac{k}{b})$;

Weibull II $[W_2(\theta,\lambda),\ \theta > 0, \lambda > 0]$

$f(x) = (\frac{\theta}{\lambda})(\frac{x}{\lambda})^{\theta-1} \exp\{-(\frac{x}{\lambda})^\theta\} \underset{(0,\infty)}{I}(x)$;

$F(x) = 1 - \exp\{-(\frac{x}{\lambda})^\theta\} \underset{(0,\infty)}{I}(x)$;

$\mathbb{E}(X^k) = \lambda^k \Gamma(1 + \frac{k}{\theta})$;

$Var(X) = \lambda^2[\Gamma(1 + \frac{2}{\theta}) - \Gamma^2(1 + \frac{1}{\theta})]$;

- $X \sim Rayleigh(\beta) \Leftrightarrow X \sim W_1(1/2\beta^2, 2)$
- $X \sim W_1(\lambda^{-1/\theta}, \theta) \Leftrightarrow X \sim W_2(\lambda, \theta)$
- $X \sim W_1(\lambda, 1) \Leftrightarrow X \sim Exp(\lambda)$
- $X \sim W_1(a, b)$ e $Y = -\ln X \Rightarrow$ $Y \sim Gumbel(b^{-1} \ln a, b^{-1})$

Normal $[N(\mu,\sigma^2),\ \mu \in \mathbb{R}, \sigma > 0]$

$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\{-\frac{(x-\mu)^2}{2\sigma^2}\} \underset{(-\infty,\infty)}{I}(x)$; $\mathbb{E}[(X - \mathbb{E}(X))^k] = \begin{cases} 0, & \text{se } k \text{ é ímpar} \\ \frac{\sigma^k k!}{2^{k/2}\Gamma(1+\frac{k}{2})}, & \text{se } k \text{ é par} \end{cases}$

$A(X) = K(X) = 0$; $M_X(t) = e^{\mu t + \sigma^2 \frac{t^2}{2}}$; $\mathbb{E}(X) = Med(X) = Mo(X) = \mu$; $Var(X) = \sigma^2$;

- $X_1, \cdots, X_n$ i.i.d. com $X_i \sim N(0,1) \Rightarrow$ $Y = \sum_{i=1}^n X_i^2 \sim \chi^2_{(n)}$
- $X \sim N(\mu, \sigma^2) \Rightarrow Y = (X - \mu)/\sigma \sim N(0,1)$
- $X_1, \cdots, X_n$ i.i.d. com $X_i \sim N(\mu, \sigma^2) \Rightarrow$ $Y = \sum_{i=1}^n a_i X_i \sim N(\mu \sum_{i=1}^n a_i, \sigma^2 \sum_{i=1}^n a_i^2)$
- X_1, X_2 i.i.d. com $X_i \sim N(0,1) \Rightarrow Y = \frac{X_1}{X_2} \sim Ca(0,1)$

Uniforme $[U(a,b),\ -\infty < a < b < \infty]$

$F(x) = \begin{cases} 0, & \text{se } x < a \\ \frac{x-a}{b-a}, & \text{se } a \le x < b \\ 1, & \text{se } x \ge b \end{cases}$;

$f(x) = \frac{1}{(b-a)} \underset{(a,b)}{I}(x)$; $\mathbb{E}(X) = Med(X) = \frac{a+b}{2}$; $Var(X) = \frac{(b-a)^2}{12}$;

$\mathbb{E}(X^k) = \sum_{i=0}^k \frac{a^{k-i}b^i}{k+1}$; $\mathbb{E}[(X - \mathbb{E}(X))^k] = \begin{cases} 0, & \text{se } k \text{ é ímpar} \\ \frac{(b-a)^k}{2^k(k+1)}, & \text{se } k \text{ é par} \end{cases}$

$M_X(t) = \frac{e^{tb} - e^{ta}}{t(b-a)}$; $A(X) = 0$; $K(X) = -\frac{6}{5}$;

- $X \sim U(0,1) \Rightarrow Y = 1 - X^{\frac{1}{n}} \sim Beta(1, n)$
- $X \sim U(0,1) \Rightarrow Y = X^2 \sim Beta(1/2, 1)$
- $X \sim U(0,1) \Rightarrow Y = -\frac{\ln X}{\lambda} \sim Exp(\lambda)$
- $X_1, \cdots, X_n$ i.i.d. com $X_i \sim U(0,1) \Rightarrow$ $Y = \min\{X_1, \cdots, X_n\} \sim Beta(1, n)$ e $Z = \max\{X_1, \cdots, X_n\} \sim Beta(n, 1)$
- $X \sim U(0,1) \Rightarrow Y = tg[\pi(x - \frac{1}{2})] \sim Ca(0,1)$

Rayleigh $[Rayleigh(\sigma),\ \sigma > 0]$

$f(x) = \frac{x}{\sigma^2} \exp\{-\frac{x^2}{2\sigma^2}\} \underset{(0,\infty)}{I}(x)$; $F(x) = 1 - \exp\{\frac{-x^2}{2\sigma^2}\} \underset{(0,\infty)}{I}(x)$; $A(X) = \frac{(\pi-3)\sqrt{\frac{\pi}{2}}}{(2-\frac{\pi}{2})^{\frac{3}{2}}}$; $Var(X) = \frac{\sigma^2(4-\pi)}{2}$;

$Med(X) = \sigma\sqrt{\ln 4}$; $\mathbb{E}(X^k) = 2^{\frac{k}{2}} \sigma^k \Gamma(\frac{k}{2} + 1)$; $\mathbb{E}(X) = \sigma\sqrt{\frac{\pi}{2}}$; $K(X) = \frac{8 - \frac{3\pi^2}{4}}{(2-\frac{\pi}{2})^2} - 3$; $Mo(X) = \sigma$;

- $X \sim Rayleigh(1) \Leftrightarrow X \sim \chi^2_{(2)}$
- $X_1, \cdots, X_n$ i.i.d. com $X_i \sim Rayleigh(\sigma) \Rightarrow$ $Y = \sum_{i=1}^n X_i^2 \sim Gama(n, 2\sigma^2)$

Pareto $[Pareto(\alpha,\beta),\ \alpha > 0, \beta > 0]$

$f(x) = \frac{\alpha\beta^\alpha}{x^{\alpha+1}} \underset{(\beta,\infty)}{I}(x)$; $F(x) = 1 - (\frac{\beta}{x})^\alpha \underset{(\beta,\infty)}{I}(x)$; $\mathbb{E}(X^k) = \frac{\alpha\beta^k}{\alpha - k}$; $Var(X) = \frac{\alpha\beta^2}{(\alpha-2)(\alpha-1)^2}$;

$x_p = \frac{\beta}{\sqrt[\alpha]{(1-p)}}$; $Med(X) = \beta\sqrt[\alpha]{2}$; $A(X) = \frac{2(\alpha+1)}{\alpha-3}\sqrt{\frac{\alpha-2}{\alpha}}$; $K(X) = \frac{6(\alpha^3+\alpha^2-6\alpha-2)}{\alpha(\alpha-3)(\alpha-4)}$; $Mo(X) = \beta$;

- $X_1, \cdots, X_n$ i.i.d. com $X_i \sim Pareto(\alpha, \beta) \Rightarrow$ $Y = \min\{X_1, \cdots, X_n\} \sim Pareto(n\alpha, \beta)$
- $X \sim Pareto(\alpha, \beta) \Rightarrow Y = \ln(\frac{X}{\beta}) \sim Exp(\alpha)$

Log – Normal $[LN(\mu,\sigma),\ \mu \in \mathbb{R},\ \sigma > 0]$

$f(x) = \frac{1}{x\sqrt{2\pi\sigma^2}} \exp\{\frac{-(\ln x - \mu)^2}{2\sigma^2}\} \underset{(0,\infty)}{I}(x)$; $\mathbb{E}(X^k) = e^{k\mu + \frac{k^2\sigma^2}{2}}$; $Var(X) = e^{2\mu+\sigma^2}(e^{\sigma^2} - 1)$;

$Med(X) = e^\mu$; $Mo(X) = e^{\mu - \sigma^2}$; $A(X) = (e^{\sigma^2} + 2)\sqrt{e^{\sigma^2} - 1}$; $K(X) = e^{4\sigma^2} + 2e^{3\sigma^2} + 3e^{2\sigma^2} - 6$;

- $X \sim LN(\mu, \sigma) \Rightarrow Y = \ln X \sim N(\mu, \sigma^2)$
- $X_1, \cdots, X_n$ i.i.d. com $X_i \sim LN(\mu, \sigma) \Rightarrow$ $Y = \prod_{i=1}^n X_i \sim LN(n\mu, \sqrt{n}\sigma)$

Cauchy $[Ca(a,b),\ a \in \mathbb{R},\ b > 0]$

$f(x) = \frac{1}{\pi b\left[1 + \left(\frac{x-a}{b}\right)^2\right]} \underset{(-\infty,\infty)}{I}(x)$; $F(x) = \frac{1}{2} + tg^{-1}(\frac{x-a}{b})$;

$Med(X) = a$; $\varphi_X(t) = e^{iat - b|t|}$; $\mathbb{E}(X^k) = indefinido$; $Mo(X) = a$;

- $X \sim Ca(a,b) \Rightarrow Y = cX + d \sim Ca(ac + d, b|c|)$
- $X_1, \cdots, X_n$ i.i.d. com $X_i \sim Ca(a_i, b_i) \Rightarrow Y = \sum_{i=1}^n X_i \sim Ca(\sum_{i=1}^n a_i, \sum_{i=1}^n b_i)$
- $X \sim Ca(0,1) \Rightarrow Y = \frac{2X}{1-X^2} \sim Ca(0,1)$
- $X \sim Ca(0,b) \Rightarrow Y = 1/X \sim Ca(0, \frac{1}{b})$

Gama $[Gama(\alpha,\beta),\ \alpha > 0,\ \beta > 0]$

$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x} \underset{(0,\infty)}{I}(x)$; $\mathbb{E}(X^k) = \frac{\Gamma(\alpha+k)}{\Gamma(\alpha)\beta^k}$; $\mathbb{E}(X) = \frac{\alpha}{\beta}$; $Var(X) = \frac{\alpha}{\beta^2}$;

$A(X) = \frac{2}{\sqrt{\alpha}}$; $K(X) = \frac{6}{\alpha}$; $M_X(t) = (\frac{\beta}{\beta - t})^\alpha,\ t < \beta$; $Mo(X) = \frac{\alpha - 1}{\beta}$;

- $X \sim Gama(\frac{n}{2}, \frac{1}{2}) \Leftrightarrow X \sim \chi^2_{(n)}$
- $X \sim Maxwell(a) \Rightarrow Y = X^2 \sim Gama(\frac{3}{2}, \frac{1}{2a^2})$
- $X_1,\ X_2$ ind. com $X_1 \sim Gama(\alpha, \beta)$ e $X_2 \sim Gama(\theta, \beta) \Rightarrow$ $Y = \frac{X_1}{X_1 + X_2} \sim Beta(\alpha, \theta)$

Beta $[Beta(a,b),\ a > 0,\ b > 0]$

$f(x) = \frac{1}{B(a,b)} x^{a-1}(1-x)^{b-1} \underset{(0,1)}{I}(x)$; $\mathbb{E}(X^k) = \frac{\Gamma(a+b)\Gamma(a+k)}{\Gamma(a+b+k)\Gamma(a)}$; $Var(X) = \frac{ab}{(a+b)^2(a+b+1)}$;

$\mathbb{E}(X) = \frac{a}{a+b}$; $Mo(X) = \frac{a-1}{a+b-2},\ a, b > 1$; $A(X) = \frac{2(b-a)\sqrt{a+b+1}}{(a+b+2)\sqrt{ab}}$; $K(X) = \frac{3(a+b+1)[2(a+b)^2 + ab(a+b+6)]}{ab(a+b+2)(a+b+3)}$;

- $X \sim Beta(a,b) \Rightarrow Y = 1 - X \sim B(b, a)$
- $X \sim Beta(a,b) \Rightarrow Y = \frac{X}{1-X} \sim Beta\ Prime(a, b)$
- $X \sim Beta(\frac{n}{2}, \frac{m}{2}) \Rightarrow Y = \frac{mX}{n(1-X)} \sim F(n, m)$
- $X \sim Beta(a, 1) \Rightarrow Y = -\ln X \sim Exp(a)$

Qui – Quadrado $[\chi^2_{(n)},\ n > 0]$

$f(x) = \frac{(\frac{1}{2})^{\frac{n}{2}}}{\Gamma(\frac{n}{2})} x^{\frac{n}{2}-1} e^{-\frac{x}{2}} \underset{(0,\infty)}{I}(x)$; $\mathbb{E}(X^k) = 2^k \frac{\Gamma(k + \frac{n}{2})}{\Gamma(\frac{n}{2})}$; $\mathbb{E}(X) = n$; $Var(X) = 2n$;

$A(X) = \sqrt{\frac{8}{n}}$; $K(X) = \frac{12}{n}$; $M_X(t) = (1 - 2t)^{-\frac{n}{2}}$; $Mo(X) = n - 2,\ n > 2$;

- $X \sim \chi^2_{(2)} \Leftrightarrow X \sim Exp(\frac{1}{2})$
- $X_1 \sim \chi^2_{(n)}$ e $X_2 \sim \chi^2_{(m)} \Rightarrow Y = \frac{X_1/n}{X_2/m} \sim F(n, m)$
- $X \sim \chi^2_{(n)}$ e $c > 0 \Rightarrow Y = \frac{X}{c} \sim Gama(\frac{n}{2}, \frac{c}{2})$
- $X_1, \cdots, X_n$ ind. com $X_i \sim \chi^2_{(n_i)} \Rightarrow Y = \sum_{i=1}^n X_i \sim \chi^2_{(\sum_{i=1}^n n_i)}$

t – Student $[t(n),\ n > 0]$

$f(x) = \frac{\Gamma(\frac{n+1}{2})}{\Gamma(\frac{n}{2})\sqrt{\pi n}} \left(1 + \frac{x^2}{n}\right)^{-\frac{(n+1)}{2}} \underset{(-\infty,\infty)}{I}(x)$; $\mathbb{E}(X^k) = \begin{cases} 0, & \text{se } k \text{ é ímpar} \\ \frac{\Gamma(\frac{k+1}{2})\Gamma(\frac{n-k}{2}) n^{\frac{k}{2}}}{\sqrt{\pi}\Gamma(\frac{n}{2})}, & \text{se } k \text{ é par} \end{cases}$

$\mathbb{E}(X) = Mo(X) = Med(X) = 0$; $Var(X) = \frac{n}{(n-2)},\ n > 2$; $A(X) = 0,\ n > 3$; $K(X) = \frac{6}{n-4},\ n > 4$;

- $X \sim t(1) \Leftrightarrow X \sim Ca(0,1)$
- $X_1 \sim N(0,1)$ e $X_2 \sim \chi^2_{(n)}$, ind. $\Rightarrow Y = \frac{X_1}{\sqrt{X_2/n}}$
- $X \sim t(n) \Rightarrow Y = X^2 \sim F(1, n)$

F – Snedecor $[F(n,m),\ n > 0,\ m > 0]$

$f(x) = \frac{n^{\frac{n}{2}} m^{\frac{m}{2}} x^{\frac{n}{2}-1}}{B(\frac{n}{2}, \frac{m}{2})(m + nx)^{\frac{n+m}{2}}} \underset{(0,\infty)}{I}(x)$; $\mathbb{E}(X^k) = \frac{(\frac{m}{n})^k \Gamma(\frac{n}{2} + k)\Gamma(\frac{m}{2} - k)}{\Gamma(\frac{n}{2})\Gamma(\frac{m}{2})}$; $Var(X) = \frac{2m^2(n+m-2)}{n(m-2)^2(m-4)},\ m > 4$;

$Mo(X) = \frac{m(n-2)}{n(m+2)}$; $A(X) = \frac{(2n+m-2)\sqrt{8(m-4)}}{(m-6)\sqrt{n(n+m-2)}},\ m > 6$; $K(X) = \frac{12[(m-2)^2(m-4) + n(n+m-2)(5m-22)]}{n(m-6)(m-8)(n+m-2)},\ m > 8$;

- $X \sim F(n,m) \Rightarrow Y = \frac{n\frac{X}{m}}{1 + n\frac{X}{m}} \sim B(\frac{n}{2}, \frac{m}{2})$
- $X \sim F(n,m) \Rightarrow Y = \lim_{m \to \infty} nX \sim \chi^2_{(n)}$
- $X \sim F(n,m) \Rightarrow Y = X^{-1} \sim F(m, n)$ e $x_p = \frac{1}{y_{1-p}}$

Laplace $[La(a,b),\ a\in \mathbb{R},\ b>0]$

$f(x)=\frac{1}{2b}e^{-\frac{|x-a|}{b}}\underset{(-\infty,\infty)}{I}(x)$; $M_X(t)=\frac{e^{at}}{1-(bt)^2},\ |t|<\frac{1}{b}$; $F(x)=\begin{cases}\frac{1}{2}e^{\frac{x-a}{b}}, & \text{se } x<a\\ 1-\frac{1}{2}e^{-\frac{x-a}{b}}, & \text{se } x\geq a\end{cases}$

$\mathbb{E}[(X-\mathbb{E}(X))^k]=\begin{cases}0, & \text{se } k \text{ é ímpar}\\ k!b^k, & \text{se } k \text{ é par}\end{cases}$ $\mathbb{E}(X)=Med(X)=Mo(X)=a$; $Var(X)=2b^2$; $A(X)=0$; $K(X)=3$;

- $\circ\ X\sim La(a,b)\Rightarrow Y=cX+d\sim La(ca+d,cb)$
- $\circ\ X\sim La(a,b)\Rightarrow Y=|X-a|\sim Exp(1/b)$
- $\circ\ X_1,X_2$ i.i.d. com $X_i\sim La(a,b)\Rightarrow Y=\frac{|X_1-a|}{|X_2-a|}\sim F(2,2)$
- $\circ\ X_1|X_2\sim N(a,X_2)$ com $X_2\sim Rayleigh(b)\Rightarrow X_1\sim La(0,1)$
- $\circ\ X_1,X_2$ i.i.d. com $X_i\sim U(0,1)\Rightarrow Y=\ln(X_1/X_2)\sim La(0,1)$

Logistica $[Logistica(a,b),\ a\in \mathbb{R},\ b>0]$

$f(x)=\frac{\exp\{-(x-a)/b\}}{b(1+\exp\{-(x-a)/b\})^2}\underset{(-\infty,\infty)}{I}(x)$; $F(X)=(1+\exp\{-(x-a)/b\})^{-1}$;

$\mathbb{E}(X)=Med(X)=Mo(X)=a$; $M_X(t)=e^{at}B(1-bt,1+bt)$;

$A(X)=0$; $K(X)=1,2$; $Var(X)=\frac{(b\pi)^2}{3}$;

- $\circ\ X\sim Logistica(a,b)\Rightarrow Y=cX+d\sim Logistica(ca+d,cb)$
- $\circ\ X\sim U(0,1)\Rightarrow Y=a+b(\ln(X)-\ln(1-X))\sim Logistica(a,b)$
- $\circ\ X\sim Exp(1)\Rightarrow Y=a-b\ln(\frac{e^{-X}}{1-e^{-X}})\sim Logistica(a,b)$
- $\circ\ X_1,X_2$ i.i.d. com $X_i\sim Exp(1)\Rightarrow Y=a-b\ln\left(\frac{X_1}{X_2}\right)\sim Logistica(a,b)$

Triangular $[Tri(a,c,b),\ \infty<a\leq c\leq b<\infty,\ a<b]$

$f(x)=\frac{2(x-a)}{(b-a)(c-a)}\underset{(a,c)}{I}(x)+\frac{2(b-x)}{(b-a)(b-c)}\underset{(c,b)}{I}(x)$; $\mathbb{E}(X)=\frac{a+b+c}{3}$; $Med(X)=\begin{cases}a+\frac{\sqrt{(b-a)(c-a)}}{\sqrt{2}}, & \text{se } c\geq\frac{a+b}{2}\\ b-\frac{\sqrt{(b-a)(b-c)}}{\sqrt{2}}, & \text{se } c\leq\frac{a+b}{2}\end{cases}$

$F(x)=\frac{(x-a)^2}{(b-a)(c-a)}\underset{(a,c)}{I}(x)+\left(1-\frac{(b-x)^2}{(b-a)(b-c)}\right)\underset{(c,b)}{I}(x)+\underset{(b,\infty)}{I}(x)$; $Var(X)=\frac{a^2+b^2+c^2-ac-ab-bc}{18}$; $Mo(X)=c$;

$M_X(t)=\frac{2(b-c)e^{at}-(b-a)e^{ct}+(c-a)e^{bt}}{t^2(b-a)(c-a)(b-c)}$; $A(X)=\frac{\sqrt{2}(a+b-2c)(2a-b-c)(a-2b+c)}{5(a^2+b^2+c^2-ab-ac-bc)^{3/2}}$; $K(X)=-\frac{3}{5}$;

- $\circ\ X_1,X_2$ i.i.d. com $X_i\sim U(0,1)\Rightarrow$ $Y=\frac{X_1+X_2}{2}\sim Tri(0,\frac{1}{2},1)$

Kumaraswamy $[Kum(a,b),\ a>0,\ b>0]$

$f(x)=abx^{a-1}(1-x^a)^{b-1}\underset{(0,1)}{I}(x)$; $F(x)=1-(1-x^a)^b\underset{(0,1)}{I}(x)+\underset{(1,\infty)}{I}(x)$; $\mathbb{E}(X^k)=bB(1+\frac{k}{a},b)$;

$Mo(X)=\left(\frac{a-1}{ab-1}\right)^{1/a}$; $Med(X)=(1-2^{-1/b})^{1/a}$; $Var(X)=bB(1+\frac{2}{a},b)-b^2B(1+\frac{1}{a},b)^2$;

- $\circ\ X\sim U(0,1)\Rightarrow Y=(1-(1-X)^{\frac{1}{b}})^{\frac{1}{a}}\sim Kum(a,b)$
- $\circ\ X\sim Kum(a,1)\Rightarrow Y=(1-X)\sim Kum(1,a)$
- $\circ\ X\sim Kum(1,1)\Leftrightarrow X\sim U(0,1)$
- $\circ\ X\sim Kum(a,1)\Rightarrow Y=-\ln(X)\sim Exp(a)$

Beta Prime $[Beta\ Prime(a,b),\ a>0,\ b>0]$

$f(x)=\frac{x^{a-1}}{B(a,b)(x+1)^{a+b}}\underset{(0,\infty)}{I}(x)$; $\mathbb{E}(X^k)=\frac{B(a+k,b-k)}{B(a,b)}$; $Var(X)=\frac{a(a+b-1)}{(b-2)(b-1)^2},\ b>2$;

$Mo(X)=\frac{a-1}{b+1},\ a\geq 1$; $A(X)=\frac{2(2a+b-1)}{b-3}\sqrt{\frac{b-2}{a(a+b-1)}},\ b>3$;

- $\circ\ X\sim Beta\ Prime(a,b)\Rightarrow Y=\frac{1}{X}\sim Beta\ Prime(b,a)$
- $\circ\ X\sim F(a,b)\Rightarrow Y=\frac{a}{b}X\sim Beta\ Prime(\frac{a}{2},\frac{b}{2})$
- $\circ\ X_1,X_2$ ind. com $X_i\sim G(a_i,1)$ $\Rightarrow Y=\frac{X_1}{X_2}\sim Beta\ Prime(a_1,a_2)$

Binomial $[Bin(n,p),\ n\in\mathbb{N},0\leq p\leq 1]$

$\mathbb{P}(X=x)=\binom{n}{x}p^x(1-p)^{n-x}\underset{\{0,\cdots,n\}}{I}(x)$; $\mathbb{E}(X)=np$; $M_X(t)=(pe^t+1-p)^n$; $\mathbb{E}\left[\frac{X!(n-k)!}{(X-k)!n!}\right]=p^k$;

$Med(X)=\begin{matrix}\lfloor np\rfloor\\ \lceil np\rceil\end{matrix}$ ou; $Mo(X)=\begin{matrix}\lfloor (n+1)p\rfloor\\ \lceil (n+1)p\rceil\end{matrix}$ ou; $Var(X)=np(1-p)$; $A(X)=\frac{1-2p}{\sqrt{np(1-p)}}$; $K(X)=\frac{1-6p(1-p)}{np(1-p)}$;

- $\circ\ X\sim Bin(1,p)\Leftrightarrow X\sim Ber(p)$
- $\circ\ X_1,\cdots,X_n$ i.i.d com $X_i\sim Ber(p)\Rightarrow$ $Y=\sum_{i=1}^n X_i\sim Bin(n,p)$
- $\circ\ X_1,\cdots,X_n$ ind com $X_i\sim Bin(n_i,p)\Rightarrow$ $Y=\sum_{i=1}^n X_i\sim Bin(\sum_{i=1}^n n_i,p)$

Geométrica I $[G_0(p),\ 0\leq p\leq 1]$

$\mathbb{P}(X=x)=p(1-p)^x\underset{\{0,1,\cdots\}}{I}(x)$; $M_X(t)=\frac{p}{1-(1-p)e^t}$;

$Mo(X)=0$; $F(x)=1-(1-p)^{x+1}$; $\mathbb{E}(X)=\frac{1-p}{p}$;

$A(X)=\frac{2-p}{\sqrt{1-p}}$; $K(X)=6+\frac{p^2}{1-p}$; $Var(X)=\frac{1-p}{p^2}$;

Geométrica II $[G_1(p),\ 0\leq p\leq 1]$

$\mathbb{P}(X=x)=p(1-p)^{x-1}\underset{\{1,2,\cdots\}}{I}(x)$; $M_X(t)=\frac{pe^t}{1-(1-p)e^t}$;

$Mo(X)=1$; $F(x)=1-(1-p)^x$; $\mathbb{E}(X)=\frac{1}{p}$;

$A(X)=\frac{2-p}{\sqrt{1-p}}$; $K(X)=\frac{6-6p+p^2}{(1-p)}$; $Var(X)=\frac{1-p}{p^2}$;

- $\circ\ X_1,\cdots,X_n$ ind. com $X_i\sim G_1(p_i)\Rightarrow$ $Y=\min_{i=1,\cdots,n}\{X_i\}\sim G_1(1-\prod_{i=1}^n(1-p_i))$
- $\circ\ X\sim Exp(\lambda)\Rightarrow Y=\lfloor X\rfloor\sim G_0(1-e^{-\lambda})$
- $\circ\ X_1,\cdots,X_r$ i.i.d. com $X_i\sim G_0(p)\Rightarrow$ $Y=\sum_{i=1}^r X_i\sim BN(r,p)$

Binomial Negativa $[BN(r,p),\ r>0,\ 0\leq p\leq 1]$

$\mathbb{P}(X=x)=\frac{\Gamma(r+x)}{\Gamma(x+1)\Gamma(r)}p^r(1-p)^x\underset{\{0,1,\cdots\}}{I}(x)$; $K(X)=\frac{p^2+6(1-p)}{r(1-p)}$;

$\mathbb{E}(X)=\frac{r(1-p)}{p}$; $Var(X)=\frac{r(1-p)}{p^2}$; $\mathbb{E}\left[\frac{X!}{(X-k)!}\right]=\frac{(r+k-1)!(1-p)^k}{(r-1)!p^k}$;

$A(X)=\frac{2-p}{\sqrt{r(1-p)}}$; $M_X(t)=\left(\frac{p}{1-(1-p)e^t}\right)^r$; $Mo(X)=\lfloor\frac{(r-1)(1-p)}{p}\rfloor$;

Pascal $[Pa(r,p),\ r\in\mathbb{N},\ 0\leq p\leq 1]$

$\mathbb{P}(X=x)=\binom{x-1}{r-1}p^r(1-p)^{x-r}\underset{\{r,r+1,\cdots\}}{I}(x)$; $A(X)=\frac{2-p}{\sqrt{r(1-p)}}$;

$\mathbb{E}\left[\frac{(X+k)!}{(X-1)!}\right]=\frac{(k+r)!}{(r-1)!p^{k+1}}$; $K(X)=\frac{6-6p+p^2}{r(1-p)}$;

$\mathbb{E}(X)=\frac{r}{p}$; $M_X(t)=\left(\frac{pe^t}{1-(1-p)e^t}\right)^r$; $Var(X)=\frac{r(1-p)}{p^2}$;

- $\circ\ X\sim Pa(1,p)\Leftrightarrow$ $X\sim G_1(p)$
- $\circ\ X\sim BN(r,p)$ e $\lambda=\frac{rp}{1-p}\Rightarrow X\sim Po(\lambda)$, quando $r\to\infty$

Uniforme Discreta $[UD(n),\ n\in\mathbb{N}]$

$\mathbb{P}(X=x)=\frac{1}{n}\underset{\{1,\cdots,n\}}{I}(x)$; $\mathbb{E}(X)=\frac{n+1}{2}$; $Var(X)=\frac{n^2-1}{12}$; $M_X(t)=\frac{e^t(1-e^{nt})}{n(1-e^t)}$; $A(X)=0$; $K(X)=-\frac{6(n^2+1)}{5(n^2-1)}$;

- $\circ\ X\sim Beta\ Binomial(n-1,1,1)$ $\Leftrightarrow X\sim UD(n)$

Poisson $[Po(\lambda),\ \lambda>0]$

$\mathbb{P}(X=x)=\frac{\lambda^x e^{-\lambda}}{x!}\underset{\{0,1,\cdots\}}{I}(x)$; $\mathbb{E}(X)=Var(X)=\lambda$; $Mo(X)=\begin{matrix}\lfloor\lambda\rfloor\\ \text{ou;}\\ \lceil\lambda\rceil-1\end{matrix}$ $A(X)=\lambda^{-1/2}$;

$\mathbb{E}[(1-a)^X]=e^{-a\lambda}$; $K(X)=\lambda^{-1}$; $M_X(t)=e^{\lambda(e^t-1)}$; $\mathbb{E}\left[\frac{X!}{(X-k)!}\right]=\lambda^k$;

- $\circ\ X_1,X_2,\cdots$ ind. com $X_i\sim Po(\frac{r^i}{i})\Rightarrow Y=\sum_{i=1}^\infty iX_i\sim G_0(\frac{1-r}{r})$
- $\circ\ X_1,\cdots,X_n$ i.i.d. com $X_i\sim Po(\lambda)\Rightarrow Y=\sum_{i=1}^n X_i\sim Po(n\lambda)$
- $\circ\ X_1,\cdots,X_n$ ind. com $X_i\sim Po(\lambda_i)\Rightarrow$ $Y=X_i|(\sum_{j=1}^n X_j=k)\sim Bin\big(k,\lambda_i/\sum_{j=1}^n\lambda_j\big)$
- $\circ\ X\sim Bin(n,p)$ e $\lambda=np\Rightarrow X\xrightarrow{\mathcal{D}}Po(\lambda)$, quando $n\to\infty$

Hipergeométrica $[Hipergeo(N,k,n),\ N\in\mathbb{N},\ k\in\{0,\cdots,N\},\ n\in\{0,\cdots,N\}]$

$\mathbb{P}(X=x)=\frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}\underset{\{\max\{0,n-(N-k)\},\cdots,\min\{k,n\}\}}{I}(x)$; $Var(X)=n\frac{k}{N}(1-\frac{k}{N})(\frac{N-n}{N-1})$;

$\mathbb{E}(X)=n\frac{k}{N}$; $Mo(X)=\lfloor\frac{(n+1)(k+1)}{N+2}\rfloor$; $A(X)=\frac{(N-2k)(N-2n)\sqrt{N-1}}{(N-2)\sqrt{nk(N-k)(N-n)}}$;

- $\circ\ X\sim Hipergeo(N,k,1)\Leftrightarrow X\sim Bernoulli(\frac{k}{N})$
- $\circ\ X\sim Hipergeo(N,k,n)$ e $p=\frac{k}{N}\Rightarrow$ $X\xrightarrow{d}Bin(n,p)$, quando $N,k\to\infty$

Apêndice

- $\circ\ Var(X)=\mathbb{E}[(X-\mathbb{E}(X))^2]$;
- $\circ\ M_X(t)=\mathbb{E}(e^{tX})$; $\varphi_X(t)=\mathbb{E}(e^{itX})$
- $\circ\ x_p=F^{-1}(p)\Rightarrow Med(X)=x_{1/2}$;
- $\circ\ A(X)=\frac{\mathbb{E}[(X-\mathbb{E}(X))^3]}{Var(X)^{3/2}}$;
  - $A(X)<0$ (cauda pesada à esquerda);
  - $A(X)>0$ (cauda pesada à direita);
  - $A(X)=0$ (distribuição simétrica);
- $\circ\ K(X)=\frac{\mathbb{E}[(X-\mathbb{E}(X))^4]}{Var(X)^2}-3$;
  - $K(X)<0$ (platicúrtica [achatada]);
  - $K(X)>0$ (leptocúrtica [afuniliada]);
  - $K(X)=0$ (mesocúrtica);

- $\circ\ e^x=\sum_{n=0}^\infty\frac{x^n}{n!}$;
- $\circ\ \ln(x+1)=\sum_{n=0}^\infty\frac{(-1)^n x^{n+1}}{n+1},\ |x|<1$;
- $\circ\ sen\ x=\sum_{n=0}^\infty\frac{(-1)^n x^{2n+1}}{(2n+1)!}$;
- $\circ\ cos\ x=\sum_{n=0}^\infty\frac{(-1)^n x^{2n}}{(2n)!}$;
- $\circ\ \sum_{i=1}^n i=\frac{n(n+1)}{2}$;
- $\circ\ \sum_{i=1}^n i^2=\frac{n(n+1)(2n+1)}{6}$;
- $\circ\ \sum_{i=1}^n i^3=\left(\frac{n(n+1)}{2}\right)^2$;
- $\circ\ \sum_{i=1}^n i^4=\frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$;
- $\circ\ \left(\sum_{i=1}^m a_i\right)^n=\sum_{x_1+\cdots+x_m=n}\frac{n!}{\prod_{j=1}^m x_j}\prod_{j=1}^m a_j^{x_j}$;

- $\circ\ \binom{n}{x}=\frac{n!}{x!(n-x)!}$;
- $\circ\ \binom{n}{x}+\binom{n}{x-1}=\binom{n+1}{x},\ n\in\mathbb{N}$ e $x\in\mathbb{Z}$;
- $\circ\ \binom{-n}{x}=(-1)^x\binom{n+x-1}{x}$;
- $\circ\ n!\sim(2\pi)^{1/2}e^{-n}n^{n+1/2}$;
- $\circ\ (a+b)^n=\sum_{i=0}^n\binom{n}{i}a^ib^{n-i}$;
- $\circ\ \binom{N_1+N_2}{n}=\sum_{i=0}^n\binom{N_1}{i}\binom{N_2}{n-i}$;
- $\circ\ (1-x)^{-2}=\sum_{i=0}^\infty(i+1)x^i$;
- $\circ\ \max(a,b)=\frac{a+b+|a-b|}{2}$;
- $\circ\ \min(a,b)=\frac{a+b-|a-b|}{2}$;

- $\circ\ \Gamma(a+1)=a\Gamma(a)$;
- $\circ\ B(a,b)=\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$;
- $\circ\ \Gamma(a)\Gamma(1-a)=\frac{\pi}{sen\ (\pi a)}$;
- $\circ\ \Gamma(\frac{a}{2})=\frac{\sqrt{\pi}(a-1)!}{2^{a-1}(\frac{a-1}{2})!}$;
- $\circ\ a_1(x-b_1)^2+a_2(x-b_2)^2$ $=(a_1+a_2)(x-c)^2$ $+\frac{a_1a_2}{a_1+a_2}(b_1-b_2)^2$, sendo $c=\frac{a_1b_1+a_2b_2}{a_1+a_2}$;
- $\circ\ e^{ix}=cos\ x+isen\ x$;
- $\circ\ e^a=\lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n$;